T.Y.B.Sc. (Mathematics)

Group Theory

Groups

(Basics)

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Groups :

Definition : Binary Operation- A binary operation x , on a set S is a function mapping

S x S into S if for each (a,b) ∈ SXS

a x b ∈ S.

Examples:

● + (usual addition) and . (usual multiplication) are binary opearation on the sets R (set of real numbers),Q(set of rational numbers),C (set of complex numbers).

Definition of Groups

- A Non - empty set G with the binary operation x is said to be group if following properties are satisfied:

  1)Closure property: for all a,b ∈ G

  implies, axb ∈ G

  2)Associative property : for all a,b,c ∈ G

  implies , a x (b x c) =(a x b) x c

  3) Existence of identity element:There is an element e in G such that for all x ∈ G

  (X x e) =(e x X)

  4)Existence of inverse element: Corresponding to each a ∈ G there is an, a' ∈ G such that,

  (a x a') = (a'x a)

Groups

Closure Property

Associative Property

Existence of Identity Element

Existence of inverse element

EXAMPLES OF GROUPS:

- 1)The set z (set of integers),R(set of real numbers),Q(set of rational numbers),C (Set of complex numbers) are all groups under usual addition +.

- 2)The set of all real valued functions with domain R under function addition is a group.

- 3)The set of nxn matrices with real entries under matrix addition is a group.

Abelian Groups:

- Definition:A group is abelian if its binary operation is commutative. i.e.

  for all $a,b \in G$, $(a \times b) = (b \times a)$

  - Examples:

  1)The set z (set of integers),R(set of real numbers),Q(set of rational numbers),C (Set of complex numbers) are all abelian groups under usual addition.

  2)The set of nxn matrices with real entries under matrix addition is an abelian group.

Elementary Properties of Groups:

- 1)Cancellation Laws holds in Groups:

Left Cancellation Law:
- a x b=a xc, Implies,b=c

Right Cancellation Law:
- b x a = c x a Implies,b=c

2)Uniqueness of Identity Element:

In a group G ,with binary operation x
there is only one element e $\in$ G Such
that,

for all x $\in$ G,

$$e \; x \; x \; = x \; \; x \; \; e$$

3)Uniqueness of inverse element:

In a group G ,with binary operation x ,for each a ∈ G there is only one element a' ∈ G Such that,

$$a \; x \; a' \; = \; a' \; x \; a \; = \; e$$

where ,e is identity element in G.

4)Reversal Law holds in Groups:

Let G be a group with binary $\times$,
for all a,b $\in$ G,

$$(a \times b)' = b' \times a'$$